Algebra 2 3.4 Long and synthetic division

Key

Obj. *Divide polynomials using long division and synthetic division; Use the Factor Theorem*

Divide $83 \div 6$

$$6\overline{)83}\quad 13$$
$$-6\downarrow$$
$$23$$
$$-18$$
$$5$$

$13\frac{5}{6}$

Watch your signs when subtract

Ex. 1A Use long division to divide x^3+5x^2+6x+9 by $(x+3)$.

$$x+3\overline{)x^3+5x^2+6x+9}\quad x^2+2x$$
$$-(x^3+3x^2)\downarrow$$
$$2x^2+6x$$
$$-(2x^2+6x)$$
$$0\quad 9 \leftarrow \text{remainder}$$

no more x

Divide lead terms $\frac{x^3}{x} = x^2$ (up top)
then multiply $x^2(x+3)$
then subtract
bring down next term and start again
$\frac{2x^2}{x} = 2x$

answer $x^2+2x+\frac{9}{x+3}$ → remainder, → divisor

1B. $8x^3+27$ by $(2x+3)$. (you will need to use placeholders)

if missing terms → $0x^2+0x$

$$2x+3\overline{)8x^3+0x^2+0x+27}\quad 4x^2-6x+9$$
$$-(8x^3+12x^2)\downarrow$$
$$-12x^2+0x$$
$$-(-12x^2-18x)$$
$$18x+27$$
$$-(18x+27)$$
$$0$$

$\frac{8x^3}{2x} = 4x^2$

$\frac{-12x^2}{2x} = -6x$

$\frac{18x}{2x} = 9$

answer $\boxed{4x^2-6x+9}$

You try. $x^3-6x^2+11x-6$ by x^2-4x+3

$$x^2-4x+3\overline{)x^3-6x^2+11x-6}\quad x-2$$
$$-(x^3-4x^2+3x)$$
$$-2x^2+8x-6$$
$$-(-2x^2+8x-6)$$
$$0$$

$\frac{x^3}{x^2} = x$

$\frac{-2x^2}{x^2} = -2$

$\boxed{x-2}$

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Synthetic division: can only use if Divide by linear term w/ coefficient of 1
ex: $x-1$ $x+1$ $x+3$ $x-3$
no → x^2 no $2x-3$

Uses coefficients only

Multiply/add

Remember

Ex. 2 Synthetic division. Divide $2x^3-7x^2-4$ by $x-3$

rewrite $2x^3-7x^2+0x-4$ w/ placeholder
write coefficients

$x-3=0$
$x=3$ opposite → 3 | 2 −7 0 −4

bring down first number
multiply/add

2	−1	−3	**−13** ← remainder
↓ ·3	↓ ·3	↓ ·3	
	6	−3	−9

Fill in x with one less power

$$2x^2-1x-3+\frac{-13}{x-3}$$

You try. Divide $3x^3-5x+10$ by $x-1$

$3x^3+0x^2-5x+10$

$x-1=0$
$x=1$ → 1 | 3 0 −5 10

3	3	−2	**8**
↓	↓	↓	
	3	3	−2

$$3x^2+3x-2+\frac{8}{x-1}$$

Remainder theorem: If P(x) is divided by x-a, the remainder is P(a)

Example 3. Use the remainder theorem to find P(-5) for $x^3+10x^2+29x+24$

−5 | 1 +10 +29 +24

1	5	4	**4**
↓	−5	−25	−20

$P(-5)=4$

Check: $(-5)^3+10(-5)^2+29(-5)+24$

$-125+250-145+24=4$

Example 4. The population of tortoises on an island is modeled by the function $P(x) = -x^3 + 6x^2 + 12x + 325$ where x is the number of years since 2015. Use the Remainder Theorem to estimate the population in 2023. $x = 8$

8 | -1 6 12 325
↓ -8 -16 -32
-1 -2 -4 | 293

293 tortoises

Using the Factor Theorem: A polynomial f(x) has a factor (x – c) if and only if f(c) = 0. (the remainder is 0)

Ex. 3 Use the Factor Theorem to determine if the binomials given are factors of $f(x)$. Use the binomials that are factors to write a factored form of $f(x)$.

Want R=0

A) $P(x) = x^4 - 8x^3 + 16x^2 - 23x - 6;\ (x - 6)$

6 | 1 -8 16 -23 -6
↓ 6 -12 24 6
1 -2 4 1 | 0 ✓

$(x-6)(x^3-2x^2+4x+1)$

B) $P(x) = x^5 - 5x^3 + 9x^2 - x + 3;\ (x+3)$ (0x⁴)

-3 | 1 0 -5 9 -1 3
↓ -3 9 -12 9 -24
1 -3 4 -3 8 | -21

not a factor

You try: $f(x) = 2x^3 - x^2 - 41x - 20;\ (x - 5),\ (x + 4)$

5 | 2 -1 -41 -20
↓ 10 45 20
2 9 4 | 0 ✓

x-5 is a factor

-4 | 2 -1 -41 -20
↓ -8 36 20
2 -9 -5 | 0

x+4 is also a Factor

(11)